**Permutation, Combination and Probability (1)**

**1.** **(a)** In how many ways can a student answers 8 true-false questions ?

 **(b)** In how many ways may the test be completed if a student is imposed for each incorrect answer, so that the student may leave some questions unanswered?

 **(a)** Number of ways = $2^{8}=256$

 **(b)** Number of ways = $3^{8}=6561$

(There are 3 choices for each question, correct, not correct, unanswered.)

**2.** In how many ways can a committee of 2 Englishmen, 2 Frenchman, 1 American be chosen from 6 Englishmen, 7 Frenchm 3 American ? In how many ways do a particular Englishman and a particular Frenchman belong to a committee ?

 Number of ways to form a committee = $C\left(6,2\right)×C\left(7,2\right)×C\left(3,1\right)=945$

Number of ways a particular Englishman and a particular Frenchman belong to a committee

 = $C\left(5,1\right)×C\left(6,1\right)×C\left(3,1\right)=90$

**3.** A company has 12 construction workers. The manager plans to assign 3 to job site A, 4 to job site B and 5 to job site C. In how many different ways can the manager make this assignment?

 **12** construction workers and each worker can only be assigned to one of the three sites.

 The number of different ways = $\frac{12!}{3!4!5!}=27720$

**4.** How many different arrangements of four letters in a row that can be made from the letters of the word (a) “COMBINE” (b) “PROBABILITY”

 **(a)** Number of different arrangements = $P\left(7,4\right)=7×6×5×4=840$

 **(b)** There are 2 ‘B’s and 2 ‘I’s. There are 11 letters. There are 7 which are not B or I.

 Ways (no B and no I) = $P\left(7,4\right)=840$

 Ways (1B, no I) = Ways(1I, no B) = $C\left(7,3\right)×4!=840$

 Ways (1B, 1I) =$ C\left(7,2\right)×4!=504$

 Ways (2B, no I) = Ways (2I, no B) = $C\left(7,2\right)×\frac{4!}{2}=252$

 Ways (2B, one I) = Ways (2I, one B) = $C\left(7,1\right)×\frac{4!}{2}=84$

 Ways (2B, 2I) = $\frac{4!}{2!2!}=6$

 Total number of ways = $840+840×2+504+252×2+84×2+6=3702$

**5.** A bag contains 5 green marbles, 4 blue marbles and 6 red marbles. A marble is picked at random. Without replacing the first marble, another marble is taken from the bag. Calculate the probability that

 **(a)** the first marble is green and the second marble red.

 **(b)** two marbles are NOT of the same colour.

 **(a)** $P\left(G\_{1}\right)=\frac{5}{5+4+6}=\frac{5}{15}=\frac{1}{3} , P\left(\left.R\_{2}\right|G\_{1}\right)=\frac{6}{4+4+6}=\frac{6}{14}=\frac{3}{7}$

 $P\left(G\_{1} and R\_{2}\right)=P\left(G\_{1}\right) P\left(\left.R\_{2}\right|G\_{1}\right)=\frac{1}{3}×\frac{3}{7}=\frac{1}{7}$

 **(b)** $P\left(two marbles are not the same colour\right)$

 $=P\left(G\_{1}\right) P\left(\left.G\_{2}\right|G\_{1}\right)+P\left(B\_{1}\right) P\left(\left.B\_{2}\right|B\_{1}\right)+P\left(R\_{1}\right) P\left(\left.R\_{2}\right|R\_{1}\right)$

 = $\frac{5}{5+4+6}×\frac{4}{4+4+6}+\frac{4}{5+4+6}×\frac{3}{5+3+6}+\frac{6}{5+4+6}×\frac{5}{5+4+5}=\frac{31}{105}$

$$P\left(two marbles are not the same colour\right)=1-P\left(two marbles are not the same colour\right)$$

 $=1-\frac{31}{105}=\frac{74}{105}$

**6.** In arranging a 10-day examination time-table involving 10 subjects and one subject per day, a teacher plans to have Mathematics, Physics and Chemistry all separated by at least one day.

How many ways are possible?

 U = universal set of all possible arrangement

 M = Mathematics, P = Physics, C = Chemistry

 MP = M and P joined in consecutive days.

 MPC = P, M and C joined in consecutive days.

 $\left|U\right|=10!$

 $\left|MP\right|=\frac{9!}{2!}$ , $\left|PC\right|=\frac{9!}{2!}$ , $\left|CM\right|=\frac{9!}{2!}$ (take MP as one subject for 2 consecutive days)

 $\left|MPC\right|=\frac{8!}{3!}$ (take MPC as one subject for 3 consecutive days)

 If P, M, C are separated by one day, possible ways

 = $\left|U\right|-\left(\left|MP\right|-\left|MPC\right|\right)-\left(\left|PC\right|-\left|MPC\right|\right)-\left(\left|CM\right|-\left|MPC\right|\right)-\left|MPC\right|$

 = $\left|U\right|-\left|MP\right|-\left|PC\right|-\left|CM\right|+2\left|MPC\right|$ = $10!-\frac{9!}{2!}-\frac{9!}{2!}-\frac{9!}{2!}+\frac{8!}{3!}=3091200$

**7.** 0000, 0001, 0002, …, 9999 are ten thousand 4-digits numbers. The numbers are classified into

 the following groups,

 **(a)** All 4 digits are the same.

 **(b)** Three digits are the same and the remaining digit is different.

 **(c)** Two pairs of the same digits

 **(d)** One pair of the same digits and the other two digits are different.

 **(e)** All digits are different.

 Calculate the number of numbers in each group.

 **(a) 10**

 **(b)** If the three same digits is 0, the other digits are 1, 2, …, 9 can be placed in unit, ten, hundred , thousand place.

 The number of numbers = $9×4$

 This is the same if the three digits is 1, 2, …, 9.

 Hence, total number of numbers = $9×4×10=360$

 **(c)** Choose any two digits from 10 digits, combination =$C\left(10,2\right)=45$

 The number of ways to place this selected digits in unit, ten, hundred , thousand place

 $=\frac{4!}{2!2!}=6$

 Hence, total number of numbers = $45×6=270$

 **(d)** There are 10 ways to choose the paired digits and there are $C\left(9,2\right)=36$ ways to choose the remaining digits.

 The number of ways to place this selected digits in unit, ten, hundred , thousand place

 $=\frac{4!}{2!}=12$

 Hence, total number of numbers = $10×36×12=4320$

 **(e)** Choose any 4 digits from 10 digits, combination =$C\left(10,4\right)=210$

 The number of ways to place this selected digits in unit, ten, hundred , thousand place

 $=4!=24$

 Hence, total number of numbers = $210×24=5040$

 **Checking :**

 **(a) + (b) + (c) + (d) + (e)**

$=C\left(10,1\right)\frac{4!}{4!}+C\left(10,1\right)C\left(9,1\right)\frac{4!}{3!1!}+C\left(10,2\right)\frac{4!}{2!2!}+C\left(10,1\right)C\left(9,2\right)\frac{4!}{2!1!1!}+C\left(10,4\right)\frac{4!}{1!1!1!1!}$

 $=10+360+270+4320+5040=10000$

**8.** A production process uses two machines in its daily production. A random sampling produced are inspected and the following contingency table is obtained

|  |  |  |
| --- | --- | --- |
|  | Defective | Non-defective |
| Machine X | 15  | 285 |
| Machine Y | 6 | 194 |

 If an item is selected randomly, what is the probability that the item is

 **(a)** defective

 **(b)** produced by machine X and defective,

 **(c)** produced by machine X or non-defective,

 **(d)** defective given that it is produced by machine X

 $P\left(\left.D\right|X\right)=\frac{15}{15+285}=\frac{1}{20}, P\left(\left.\overbar{D}\right|X\right)=\frac{285}{15+285}=\frac{19}{20}, $

 $P\left(\left.D\right|Y\right)=\frac{6}{6+194}=\frac{3}{100}, ,P\left(\left.\overbar{D}\right|Y\right)=\frac{194}{6+194}=\frac{97}{100} $

 **(a)** $P\left(\overbar{D}\right)=P\left(X\right)P\left(\left.\overbar{D}\right|X\right)+P\left(Y\right)P\left(\left.\overbar{D}\right|Y\right)=\frac{1}{2}×\frac{19}{20}+\frac{1}{2}×\frac{97}{100}=\frac{24}{25}$

 **(b)** $P\left(X and \overbar{D}\right)=P\left(X\right)P\left(\left.\overbar{D}\right|X\right)=\frac{1}{2}×\frac{19}{20}=\frac{19}{40}$

 **(c)** **Method 1**

 $P\left(Y and \overbar{D}\right)=P\left(Y\right)P\left(\left.\overbar{D}\right|Y\right)=\frac{1}{2}×\frac{97}{100}=\frac{97}{200}$

 $P\left(X and D\right)=P\left(X\right)P\left(\left.D\right|X\right)=\frac{1}{2}×\frac{1}{20}=\frac{1}{40}$

 $P\left(Y and D\right)=P\left(Y\right)P\left(\left.D\right|Y\right)=\frac{1}{2}×\frac{3}{100}=\frac{3}{200}$

 $P\left(X or \overbar{D}\right)=P\left(X and \overbar{D}\right)+P\left(X and D\right)+P\left(Y and \overbar{D}\right)=\frac{19}{40}+\frac{1}{40}+\frac{97}{200}=\frac{197}{200}=0.985$

 **Method 2**

 $P\left(X and D\right)=P\left(X\right)P\left(\left.D\right|X\right)=\frac{1}{2}×\frac{1}{20}=\frac{1}{40}$

 $P\left(X or \overbar{D}\right)=P\left(\overbar{D}\right)+P\left(X and D\right)=\frac{24}{25}+\frac{1}{40}=\frac{197}{200}=0.985$

 **Method 3**

 $P\left(X or \overbar{D}\right)=P\left(X\right)+P\left(\overbar{D}\right)-P\left(X and \overbar{D}\right)=P\left(X\right)+P\left(\overbar{D}\right)-P\left(X\right)P\left(\left.\overbar{D}\right|X\right)$

 $=\frac{1}{2}+\frac{24}{25}-\frac{1}{2}×\frac{19}{20}=\frac{197}{200}=0.985$

 **(d)** $P\left(\left.D\right|X\right)=\frac{1}{20}$

**9.** A navigation signal is made of flags arranged in a row. If there are 4 red flags, 2 blue flags and 2 green flags, find the number of different signals possible if

 **(a)** we can use all the flags

 **(b)** at least 7 flags must be used for the signal.

 **(a)** Number of different signals possible = $\frac{\left(4+2+2\right)!}{4!2!2!}=420$

 **(b)** Number of different signals possible if 7 flags are used

 = $Arrange\left(4R,2B,1G\right)+Arrange\left(4R,1B,2G\right)+Arrange\left(4R,2B,1G\right)+Arrange\left(3R,2B,2G\right)$

 = $\frac{\left(4+2+1\right)!}{4!2!1!}+\frac{\left(4+1+2\right)!}{4!1!2!}+\frac{\left(3+2+2\right)!}{3!2!2!}=420$

 The number of different signals possible if at least 7 flags must be used for the signal

 = $420+424=840$

**10.** A group of students sit for both the Mathematics and Physics papers in school examination. Their results are summarized as follow:

 75% pass Mathematics

 70% passes Physics

 40% fail in at least one of the subjects.

 A student is selected randomly from the group.

 **(a)** Find the probability that the student passes only one of the two subjects.

 **(b)** Among those who pass Mathematics, find the probability that they also pass Physics.

 **(a)** $P\left(M\right)=0.75, P\left(P\right)=0.7, P\left(M^{'}∪P^{'}\right)=0.4$

 $P\left(M^{'}∪P^{'}\right)=P\left(\left(M∩P\right)'\right)⟹P\left(M∩P\right)=1-0.4=0.6$

 $P\left(M ∆ P\right)=P\left(M\right)+P\left(P\right)-2P\left(M∩P\right)=0.75+0.7-2\left(0.6\right)=0.25$

 **(b)** $P\left(P|M\right)=\frac{P\left(P∩M\right)}{P(M)}=\frac{0.6}{0.75}=0.8$

**11.** In how many ways can a committee of 3 women and 4 men are chosen from 8 women and 7 men? What is the number of ways if Miss X refuses to serve if Mr. Y is a member?

 Number of ways to form a committee = $C\left(8,3\right)×C\left(7,4\right)=56×35=1960$

 Number of ways if Miss X refuses to serve if Mr. Y is a member

 **=** $Ways\left(X∩Y^{'}\right)+Ways\left(X'∩Y\right)+Ways\left(X'∩Y^{'}\right)$

 = $C\left(7,2\right)×C\left(6,4\right)+C\left(7,3\right)×C\left(6,3\right)+C\left(7,3\right)×C\left(6,4\right)=1540$

**12.** Find the number of permutations that can be formed from the letters of the word POPULAR. How many of these permutations:

 **(a)** begin and end with P?

 **(b)** have the two P’s separated?

 **(c)** have the vowels together?

 **(a)** Since the P’s are fixed, the other 5 letters can be permutated.

 Number of permutations = $5!=120$

 **(b)** If the two P’s must be placed together, let this two P’s are joined as one letter (PP), so the number of permutations = $\left(7-1\right)!=720$

 Total permutations with the two P’s joined or not joined together = $\frac{7!}{2!}=2520$

 Number of permutations with the two P’s separated = $2520-720=1800$

 **(c)** There are 3 vowels O,U,A , let them joined together as one letter (OUA), so there are 5 letters

 {(OUA), P,P, L, R} , number of permutations = $\frac{5!}{2!}=60$

 However the vowels O,U,A can be permutated and the number of permutations = $3!=6$

 So the number of permutations = $60×6=360$

**Yue Kwok Choy**

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